Исходная система

$$\begin{cases} u_t(x,t) = Du_{xx}(x,t) & x \in (0,l) & t \in (0,T] \\ u(x,0) = \varphi(x) & x \in [0,l] & t = 0 \\ u_x(0,t) - Hu(0,t) = 0 & x = 0 & t \in (0,T] \\ u_x(l,t) + Hu(l,t) = 0 & x = l & t \in (0,T] \end{cases}$$

Простейшая явная схема

$$\begin{cases} \dfrac{u_i^k - u_i^{k-1}}{h_t} = D\dfrac{u_{i+1}^{k-1} - 2u_i^{k-1} + u_{i-1}^{k-1}}{h_x^2} & i = \overline{1, I-1} \quad k = \overline{1, K} \\ u_i^0 = \varphi_i & i = \overline{0, I} \quad k = 0 \\ \dfrac{u_1^k - u_0^k}{h_x} - Hu_0^k = 0 & i = 0 \quad k = \overline{1, K} \\ \dfrac{u_I^k - u_{I-1}^k}{h_x} + Hu_I^k = 0 & i = I \quad k = \overline{1, K} \end{cases}$$

Расчет

$$\begin{cases} u_i^k = \gamma u_{i+1}^{k-1} + (1-2\gamma)u_i^{k-1} + \gamma u_{i-1}^{k-1} & i = \overline{1, I-1} \quad k = \overline{1, K} \\ u_i^0 = \varphi_i & i = \overline{0, I} \quad k = 0 \\ u_0^k = \dfrac{u_1^k}{Hh_x + 1} & i = 0 \quad k = \overline{1, K} \\ u_I^k = \dfrac{u_{I-1}^k}{Hh_x + 1} & i = I \quad k = \overline{1, K} \end{cases}$$

Простейшая неявная схема

$$\begin{cases} \dfrac{u_i^k - u_i^{k-1}}{h_t} = D\dfrac{u_{i+1}^k - 2u_i^k + u_{i-1}^k}{h_x^2} & i = \overline{1, I-1} \quad k = \overline{1, K} \\ u_i^0 = \varphi_i & i = \overline{0, I} \quad k = 0 \\ \dfrac{u_1^k - u_0^k}{h_x} - Hu_0^k = 0 & i = 0 \quad k = \overline{1, K} \\ \dfrac{u_I^k - u_{I-1}^k}{h_x} + Hu_I^k = 0 & i = I \quad k = \overline{1, K} \end{cases}$$

Расчет

$$\begin{cases} \gamma u_{i-1}^k + (-2\gamma - 1)u_i^k + \gamma u_{i+1}^k = -u_i^{k-1} & i = \overline{1, I-1} \quad k = \overline{1, K} \\ u_i^0 = \varphi_i & i = \overline{0, I} \quad k = 0 \\ (Hh_x + 1)u_0^k - u_1^k = 0 & i = 0 \quad k = \overline{1, K} \\ -u_{I-1}^k + (Hh_x + 1)u_I^k = 0 & i = I \quad k = \overline{1, K} \end{cases}$$

Схема Кранка-Николсон

$$\begin{cases} \dfrac{u_i^k - u_i^{k-1}}{h_t} = \dfrac{D}{2}\dfrac{\left(u_{i+1}^k - 2u_i^k + u_{i-1}^k\right) + \left(u_{i+1}^{k+1} - 2u_i^{k+1} + u_{i-1}^{k+1}\right)}{h_x^2} & i = \overline{1, I-1} \quad k = \overline{1, K} \\ u_i^0 = \varphi_i & i = \overline{0, I} \quad k = 0 \\ \dfrac{u_1^k - u_0^k}{h_x} - Hu_0^k = 0 & i = 0 \quad k = \overline{1, K} \\ \dfrac{u_I^k - u_{I-1}^k}{h_x} + Hu_I^k = 0 & i = I \quad k = \overline{1, K} \end{cases}$$

Расчет

$$\begin{cases} \gamma u_{i-1}^k + \left(-2\gamma - 2\right)u_i^k + \gamma u_{i+1}^k = -\gamma u_{i-1}^{k-1} + \left(2\gamma - 2\right)u_i^{k-1} - \gamma u_{i+1}^{k-1} & i = \overline{1, I-1} \quad k = \overline{1, K} \\ u_i^0 = \varphi_i & i = \overline{0, I} \quad k = 0 \\ \left(Hh_x + 1\right)u_0^k - u_1^k = 0 & i = 0 \quad k = \overline{1, K} \\ -u_{I-1}^k + \left(Hh_x + 1\right)u_I^k = 0 & i = I \quad k = \overline{1, K} \end{cases}$$

Простейшая неявная схема повышенного порядка аппроксимации

$$\begin{cases} \dfrac{u_i^k - u_i^{k-1}}{h_t} = D\dfrac{u_{i+1}^k - 2u_i^k + u_{i-1}^k}{h_x^2} & i = \overline{0,I} \quad k = \overline{1,K} \\ u_i^0 = \varphi_i & i = \overline{0,I} \quad k = 0 \\ \dfrac{u_1^k - u_{-1}^k}{2h_x} - Hu_0^k = 0 & i = 0 \quad k = \overline{1,K} \\ \dfrac{u_{I+1}^k - u_{I-1}^k}{2h_x} + Hu_I^k = 0 & i = I \quad k = \overline{1,K} \end{cases}$$

Расчет

$$\begin{cases} \gamma u_{i-1}^k + (-2\gamma - 1)u_i^k + \gamma u_{i+1}^k = -u_i^{k-1} & i = \overline{1,I-1} \quad k = \overline{1,K} \\ u_i^0 = \varphi_i & i = \overline{0,I} \quad k = 0 \\ (2H\gamma h_x + 2\gamma + 1)u_0^k - 2\gamma u_1^k = u_0^{k-1} & i = 0 \quad k = \overline{1,K} \\ -2\gamma u_{I-1}^k + (2H\gamma h_x + 2\gamma + 1)u_I^k = u_I^{k-1} & i = I \quad k = \overline{1,K} \end{cases}$$

Схема Кранка-Николсон повышенного порядка аппроксимации

$$\begin{cases} \dfrac{u_i^k - u_i^{k-1}}{h_t} = \dfrac{D}{2}\dfrac{\left(u_{i+1}^k - 2u_i^k + u_{i-1}^k\right) + \left(u_{i+1}^{k+1} - 2u_i^{k+1} + u_{i-1}^{k+1}\right)}{h_x^2} & i = \overline{0,I} \quad k = \overline{1,K} \\ u_i^0 = \varphi_i & i = \overline{0,I} \quad k = 0 \\ \dfrac{u_1^k - u_{-1}^k}{2h_x} - Hu_0^k = 0 & i = 0 \quad k = \overline{1,K} \\ \dfrac{u_{I+1}^k - u_{I-1}^k}{2h_x} + Hu_I^k = 0 & i = I \quad k = \overline{1,K} \end{cases}$$

Расчет

$$\begin{cases} \gamma u_{i-1}^k + \left(-2\gamma - 2\right)u_i^k + \gamma u_{i+1}^k = -\gamma u_{i-1}^{k-1} + \left(2\gamma - 2\right)u_i^{k-1} - \gamma u_{i+1}^{k-1} & i = \overline{1,I-1} \quad k = \overline{1,K} \\ u_i^0 = \varphi_i & i = \overline{0,I} \quad k = 0 \\ \left(H\gamma h_x + \gamma + 1\right)u_0^k - \gamma u_1^k = \left(-H\gamma h_x - \gamma + 1\right)u_0^{k-1} + \gamma u_1^{k-1} & i = 0 \quad k = \overline{1,K} \\ -\gamma u_{I-1}^k + \left(H\gamma h_x + \gamma + 1\right)u_I^k = \gamma u_{I-1}^{k-1} + \left(-H\gamma h_x - \gamma + 1\right)u_I^{k-1} & i = I \quad k = \overline{1,K} \end{cases}$$